



US009189729B2

(12) **United States Patent**  
**Arthur et al.**

(10) **Patent No.:** **US 9,189,729 B2**  
(45) **Date of Patent:** **Nov. 17, 2015**

(54) **SCALABLE NEURAL HARDWARE FOR THE NOISY-OR MODEL OF BAYESIAN NETWORKS**

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(\*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 304 days.

(21) Appl. No.: **13/562,187**

(22) Filed: **Jul. 30, 2012**

(65) **Prior Publication Data**

US 2015/0286924 A1 Oct. 8, 2015

(51) **Int. Cl.**  
**G06N 3/04** (2006.01)  
**G06N 3/08** (2006.01)  
**G06F 7/58** (2006.01)  
**G06N 7/00** (2006.01)

(52) **U.S. Cl.**  
CPC **G06N 3/08** (2013.01); **G06F 7/582** (2013.01);  
**G06N 3/04** (2013.01); **G06N 7/005** (2013.01)

(58) **Field of Classification Search**  
CPC ..... G06N 3/04; G06N 3/06; G06N 3/0472;  
G06N 3/0481  
See application file for complete search history.

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*Primary Examiner* — Jeffrey A Gaffin

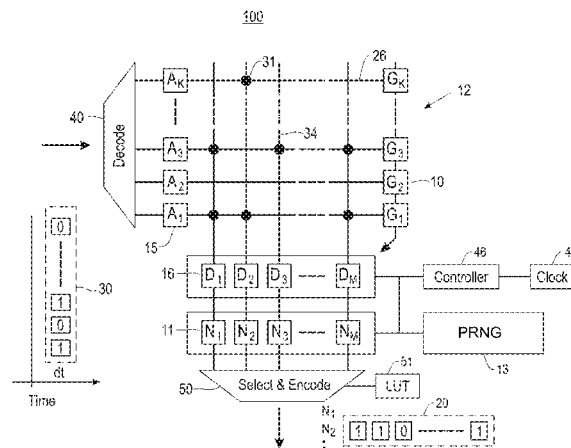
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(57) **ABSTRACT**

Embodiments of the invention relate to a scalable neural hardware for the noisy-OR model of Bayesian networks. One embodiment comprises a neural core circuit including a pseudo-random number generator for generating random numbers. The neural core circuit further comprises a plurality of incoming electronic axons, a plurality of neural modules, and a plurality of electronic synapses interconnecting the axons to the neural modules. Each synapse interconnects an axon with a neural module. Each neural module receives incoming spikes from interconnected axons. Each neural module represents a noisy-OR gate. Each neural module spikes probabilistically based on at least one random number generated by the pseudo-random number generator unit.

**20 Claims, 11 Drawing Sheets**



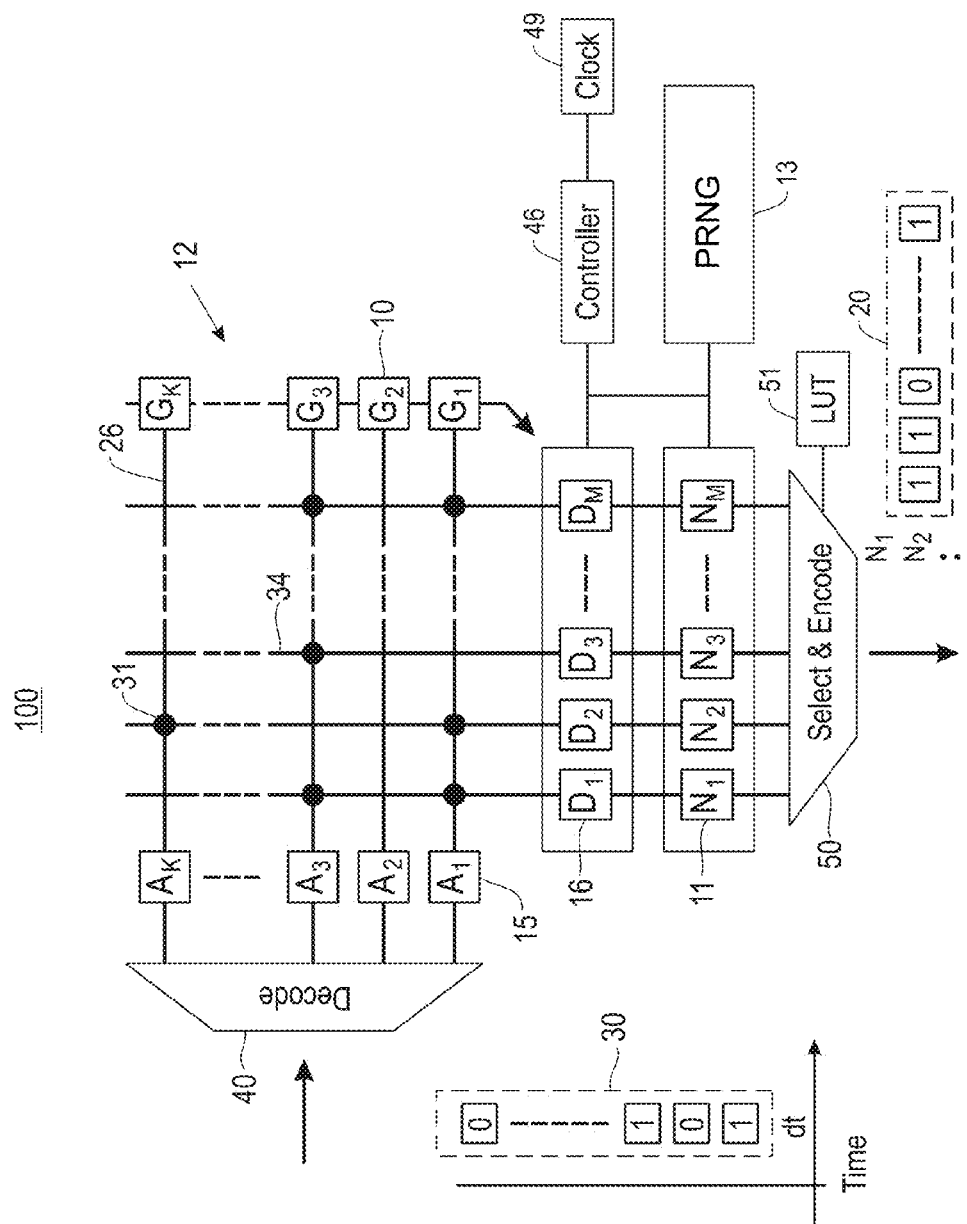
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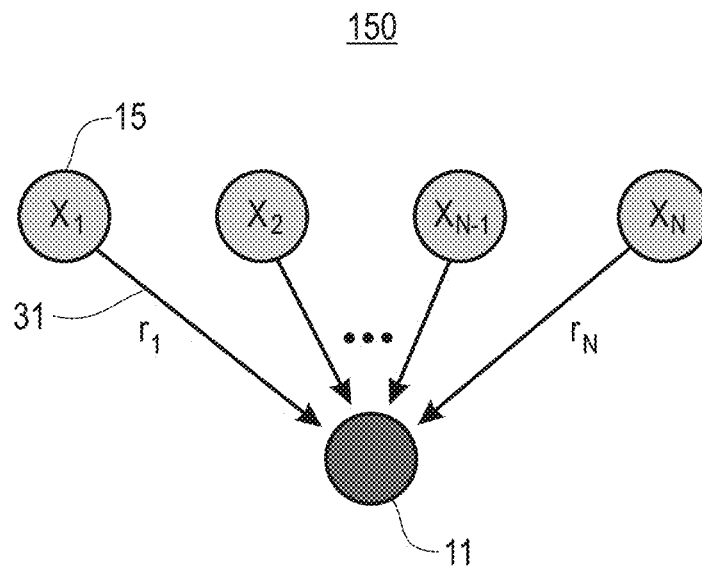


FIG. 2

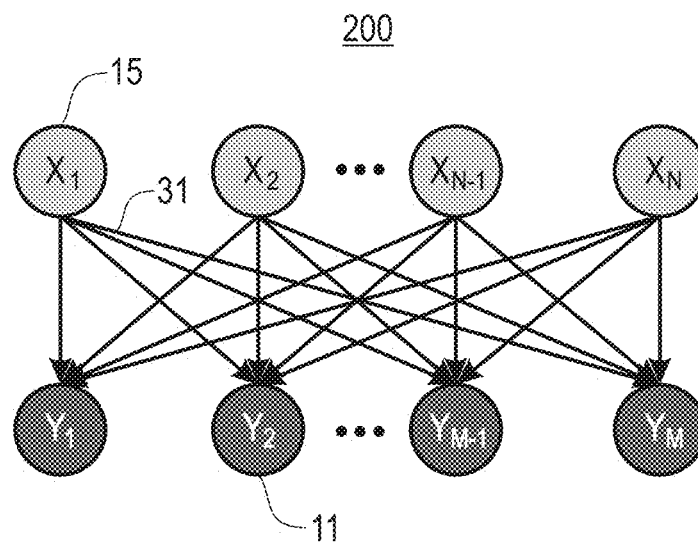


FIG. 3

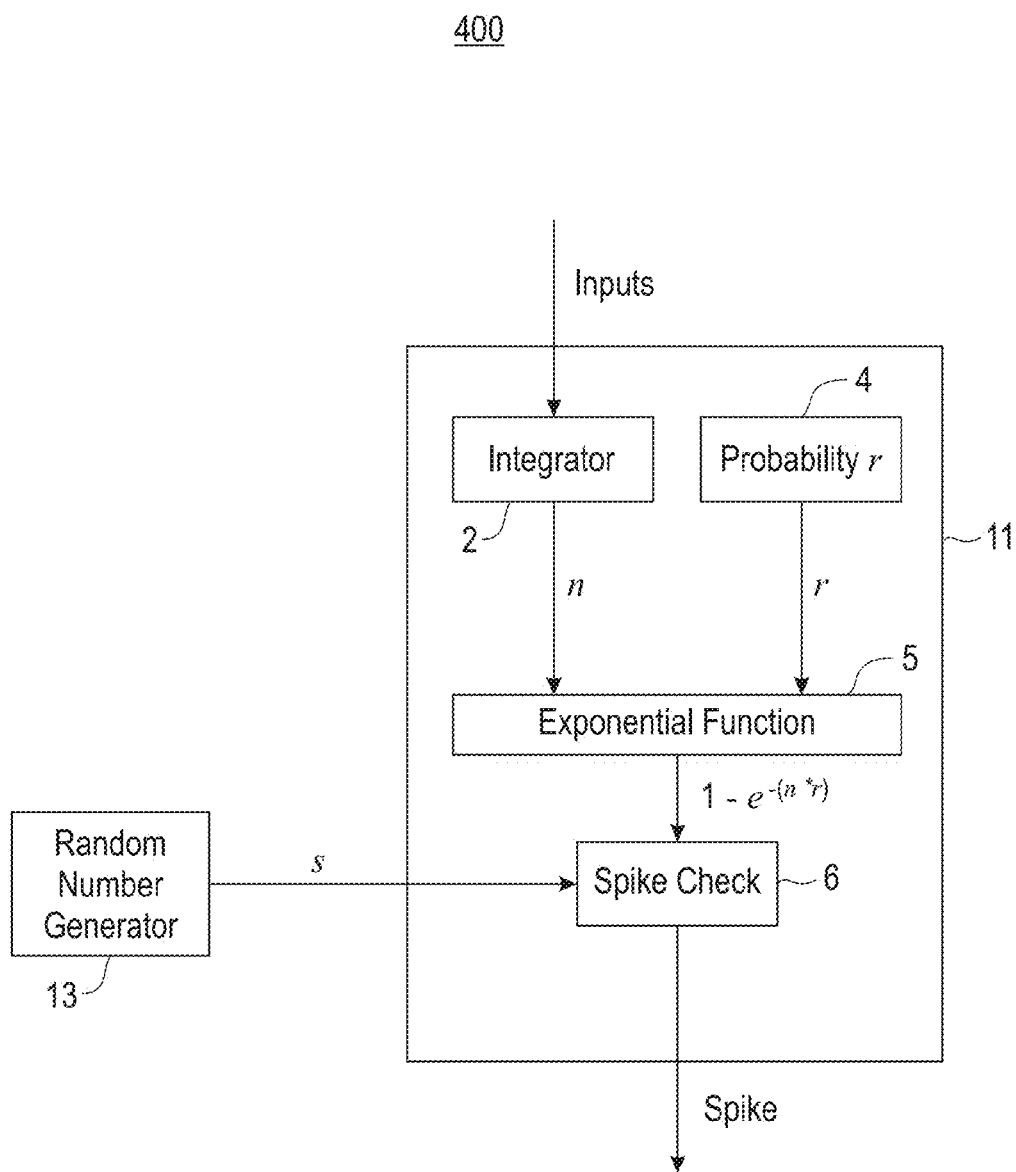


FIG. 4

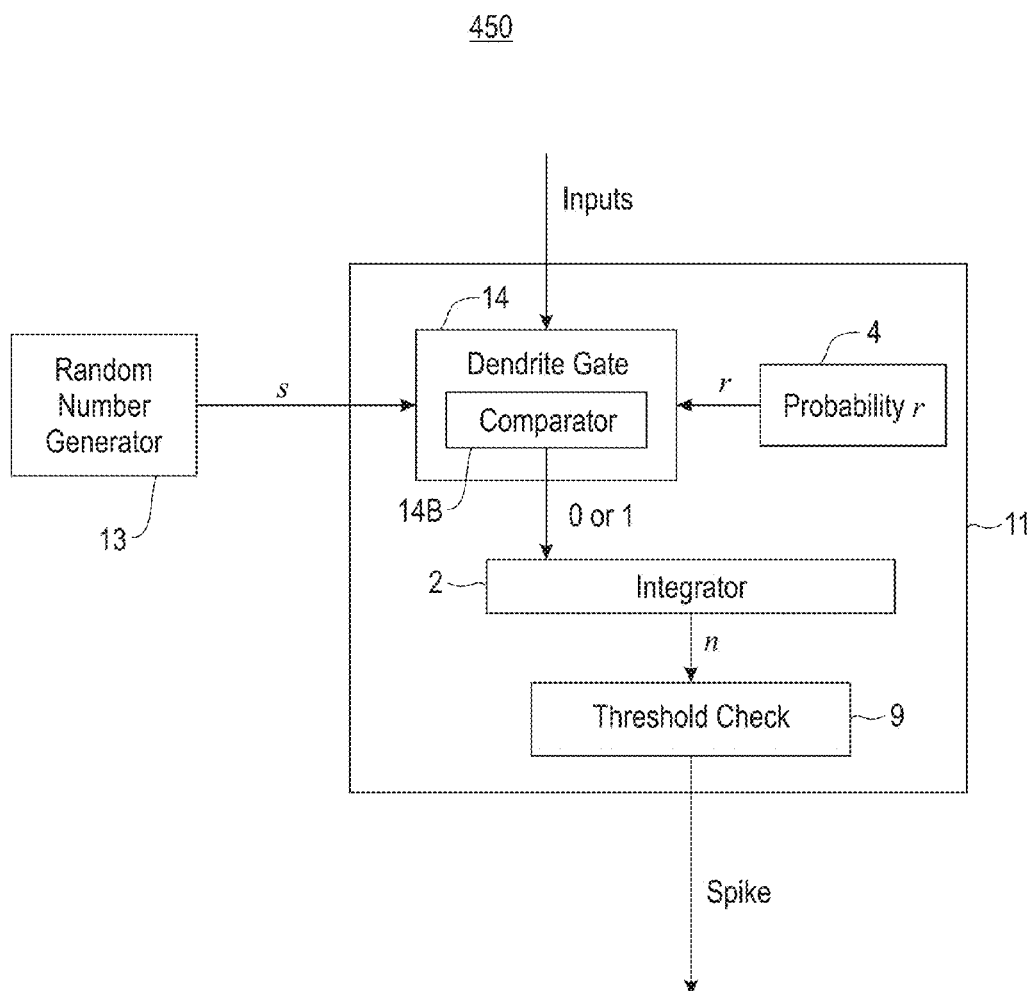


FIG. 5

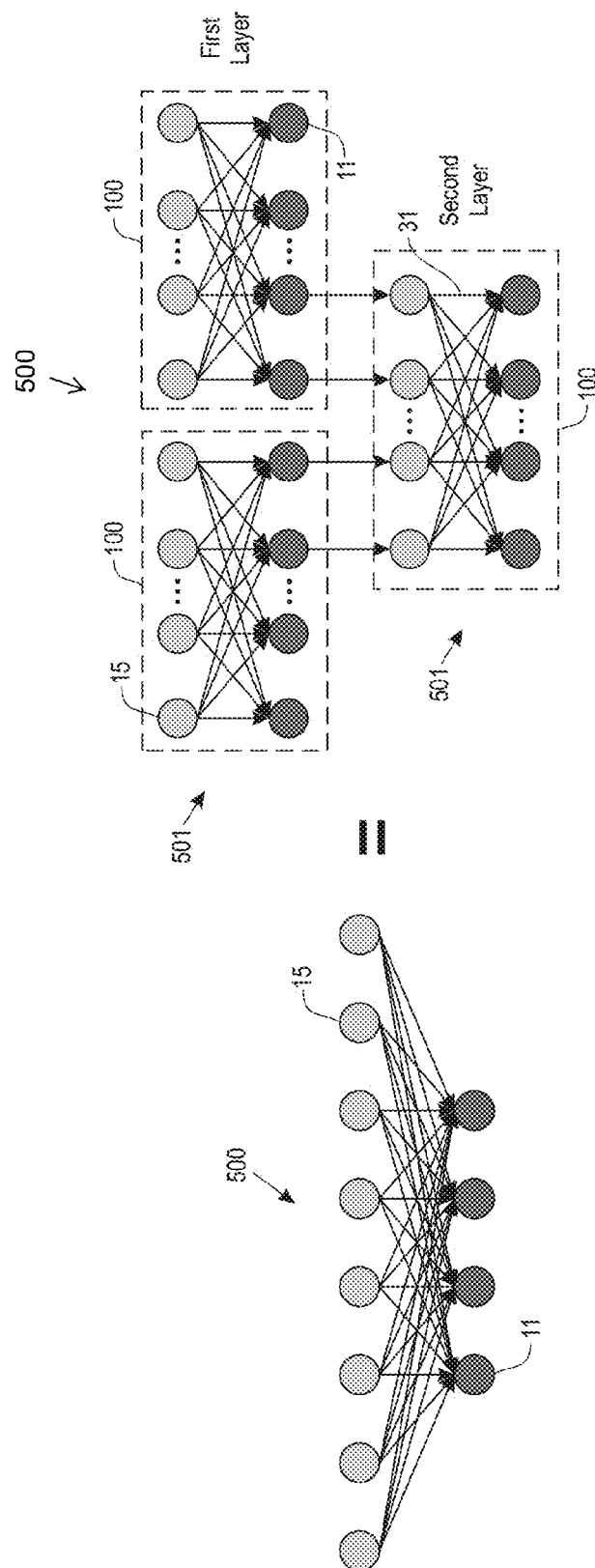


FIG. 6

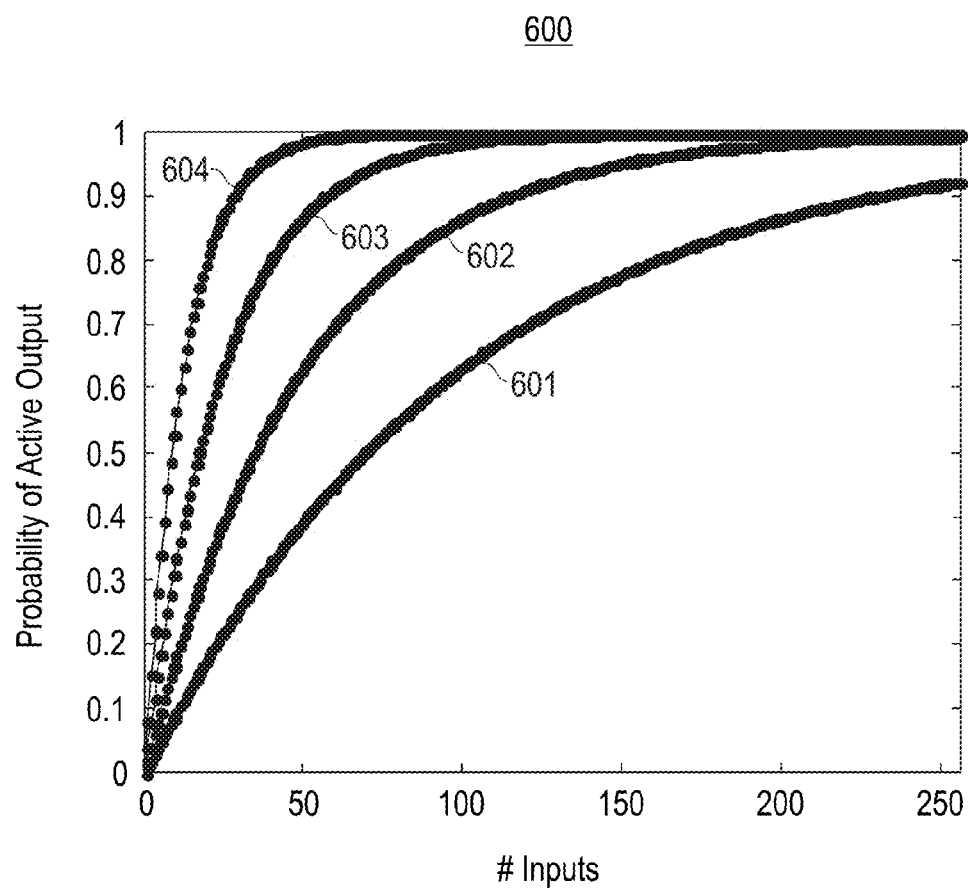
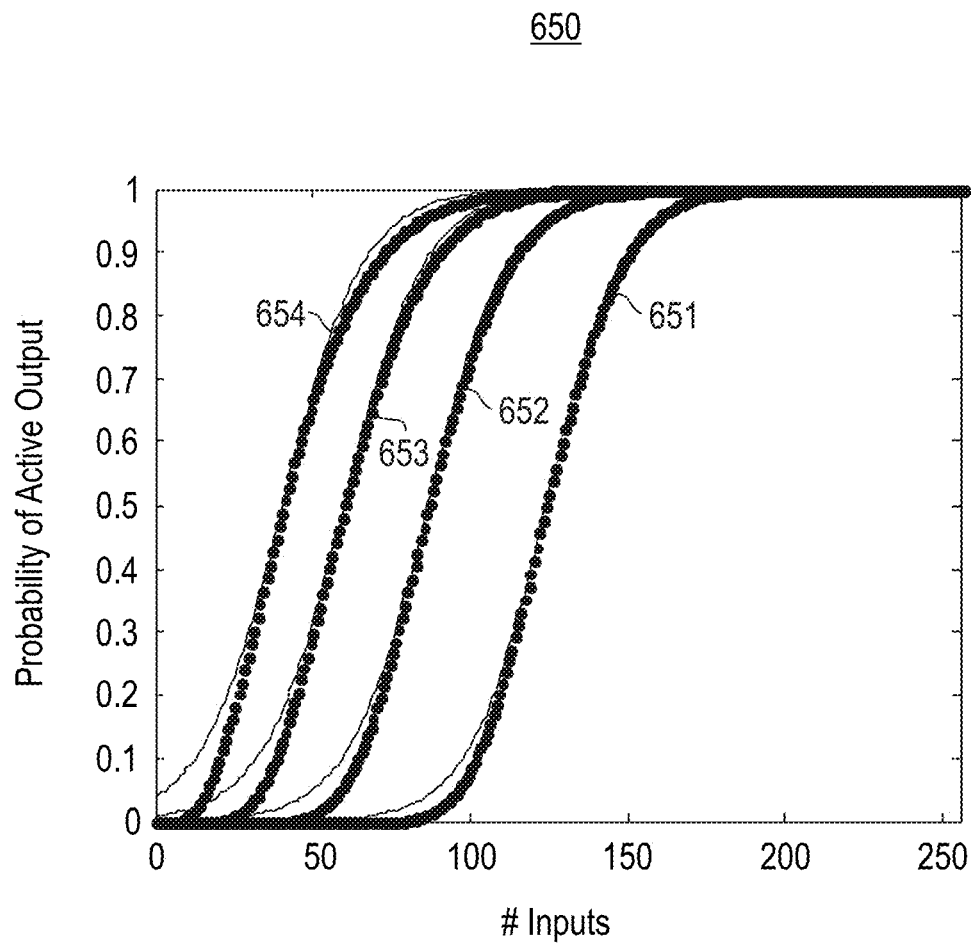
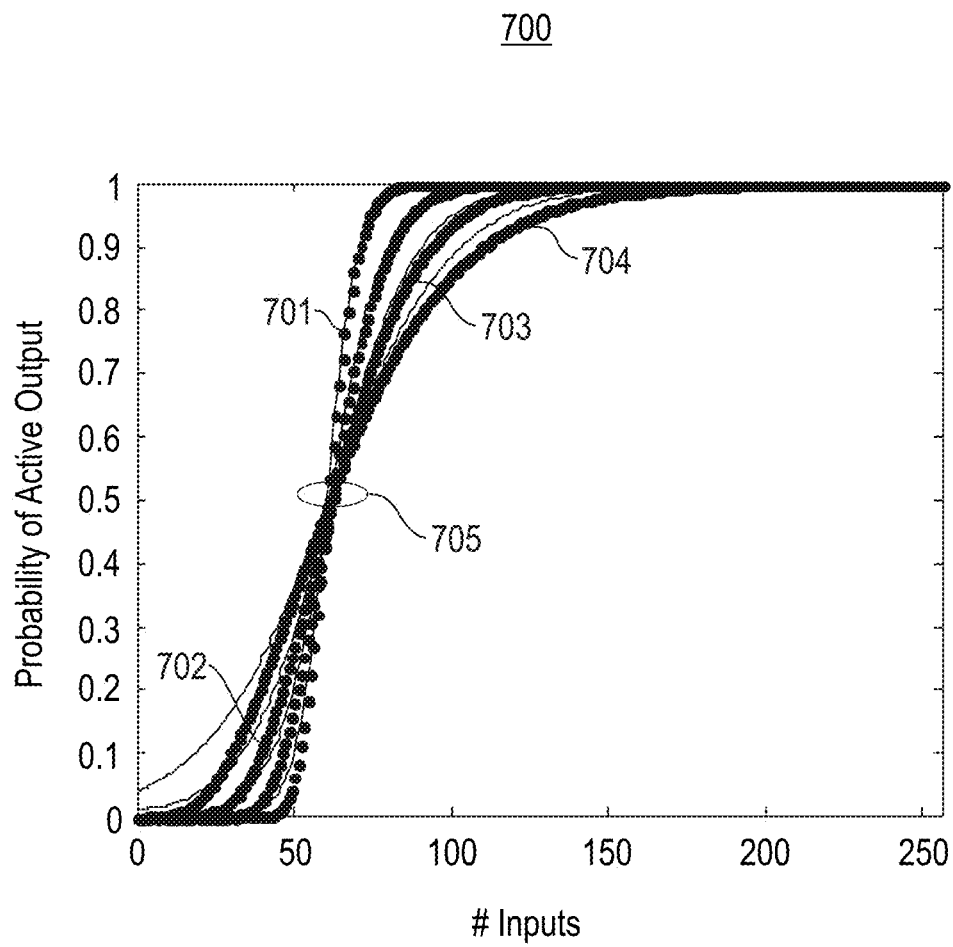


FIG. 7



**FIG. 8**

**FIG. 9**

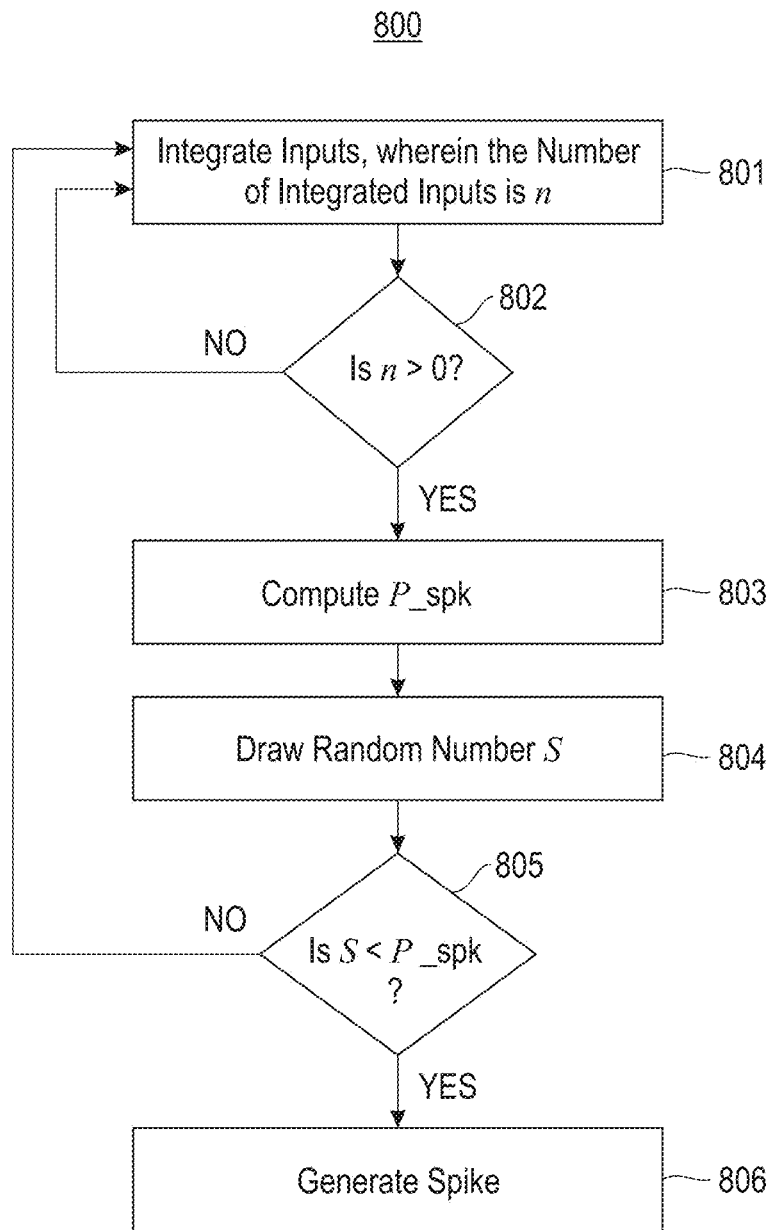


FIG. 10

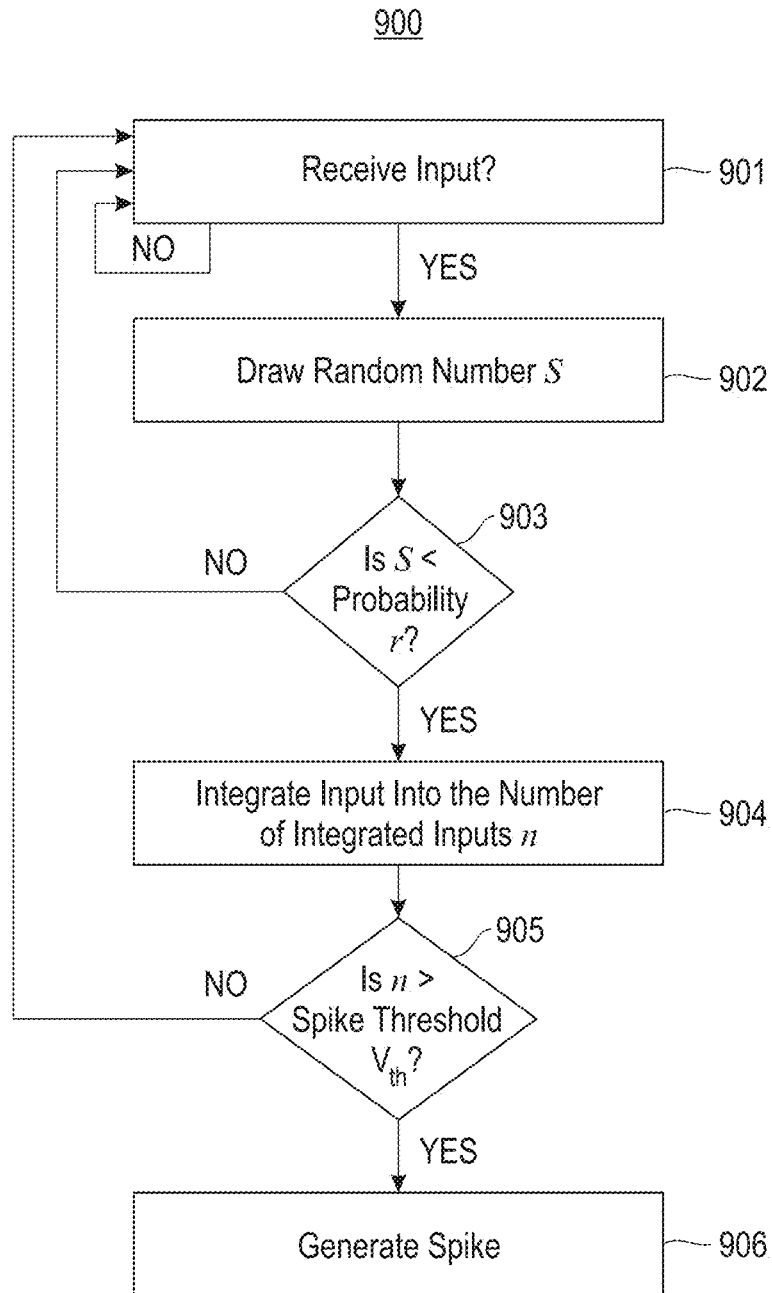


FIG. 11

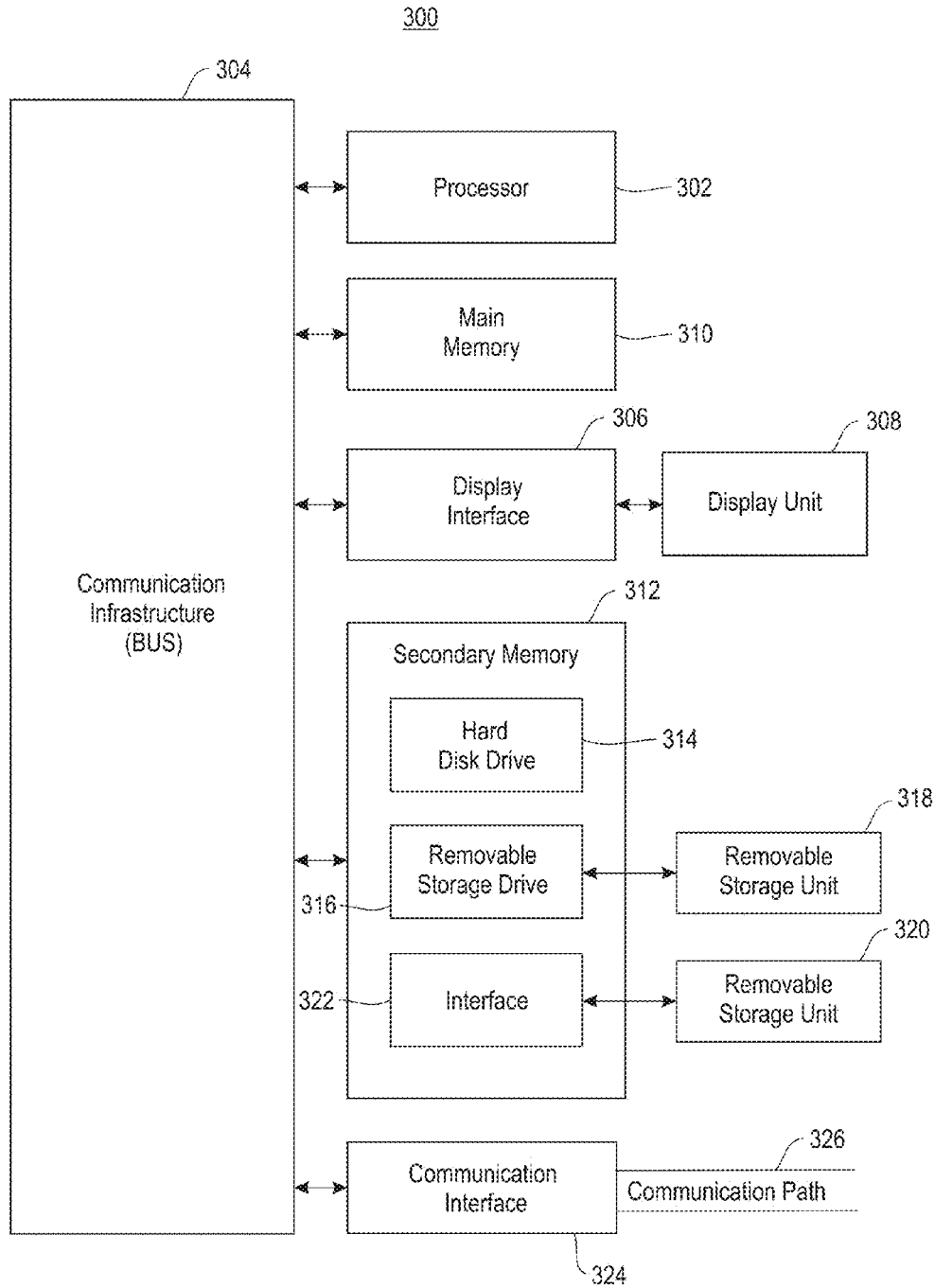


FIG. 12

# SCALABLE NEURAL HARDWARE FOR THE NOISY-OR MODEL OF BAYESIAN NETWORKS

This invention was made with Government support under HR0011-09-C-0002 awarded by Defense Advanced Research Projects Agency (DARPA). The Government has certain rights in this invention.

## BACKGROUND

Embodiments of the invention relate to neuromorphic and synaptronic computation, and in particular, a scalable neural hardware for the noisy-OR model of Bayesian networks.

Neuromorphic and synaptronic computation, also referred to as artificial neural networks, are computational systems that permit electronic systems to essentially function in a manner analogous to that of biological brains. Neuromorphic and synaptronic computation do not generally utilize the traditional digital model of manipulating 0s and 1s. Instead, neuromorphic and synaptronic computation create connections between processing elements that are roughly functionally equivalent to neurons of a biological brain. Neuromorphic and synaptronic computation may comprise various electronic circuits that are modeled on biological neurons.

In biological systems, the point of contact between an axon of a neuron and a dendrite on another neuron is called a synapse, and with respect to the synapse, the two neurons are respectively called pre-synaptic and post-synaptic. The essence of our individual experiences is stored in conductance of the synapses. The synaptic conductance changes with time as a function of the relative spike times of pre-synaptic and post-synaptic neurons, as per spike-timing dependent plasticity (STDP). The STDP rule increases the conductance of a synapse if its post-synaptic neuron fires after its pre-synaptic neuron fires, and decreases the conductance of a synapse if the order of the two firings is reversed.

## BRIEF SUMMARY

Embodiments of the invention relate to a scalable neural hardware for the noisy-OR model of Bayesian networks. One embodiment comprises a neural core circuit including a pseudo-random number generator for generating random numbers. The neural core circuit further comprises a plurality of incoming electronic axons, a plurality of neural modules, and a plurality of electronic synapses interconnecting the axons to the neural modules. Each synapse interconnects an axon with a neural module. Each neural module receives incoming spikes from interconnected axons. Each neural module represents a noisy-OR gate. Each neural module spikes probabilistically based on at least one random number generated by the pseudo-random number generator.

Another embodiment comprises receiving one or more incoming spikes from one or more incoming axons in a neural network, and probabilistically generating an outgoing spike in response to said one or more incoming spikes. The outgoing spike is probabilistically generated based on or more random numbers using a noisy-OR gate model.

These and other features, aspects and advantages of the present invention will become understood with reference to the following description, appended claims and accompanying figures.

## BRIEF DESCRIPTION OF THE SEVERAL VIEWS OF THE DRAWINGS

FIG. 1 shows a neural core circuit, in accordance with an embodiment of the invention;

FIG. 2 shows a noisy-OR system including only one noisy-OR neural module, in accordance with an embodiment of the invention;

FIG. 3 shows a noisy-OR system including multiple noisy-OR neural modules, in accordance with an embodiment of the invention;

FIG. 4 is a block diagram showing a neuron computation circuit for a noisy-OR neural module, wherein the neuron computation circuit is configured to compute an exponential function, in accordance with an embodiment of the invention;

FIG. 5 is a block diagram showing a neuron computation circuit for a noisy-OR neural module, wherein the neuron computation circuit includes a dendrite gate unit, in accordance with an embodiment of the invention;

FIG. 6 shows a scalable noisy-OR neural network, in accordance with an embodiment of the invention;

FIG. 7 is an example graph plotting the spiking probabilities of different example noisy-OR neural modules, wherein each neural module has a different probability  $r$ , in accordance with an embodiment of the invention;

FIG. 8 is an example graph plotting the spiking probabilities of different example noisy-OR neural modules, wherein each neural module has a different spiking threshold  $V_{th}$ , in accordance with an embodiment of the invention;

FIG. 9 is an example graph plotting the spiking probabilities of different example noisy-OR neural modules, wherein each neural module has a different spiking threshold  $V_{th}$  and maintains a different probability  $r$ , in accordance with an embodiment of the invention;

FIG. 10 is a flowchart of an example process for implementing probabilistic spiking in a neural module, wherein the process includes computing an exponential function, in accordance with an embodiment of the invention;

FIG. 11 is a flowchart of an example process for implementing probabilistic spiking in a neural module, wherein the process includes determining whether the number of inputs integrated in the neural module is greater than a spiking threshold of the neural module, in accordance with an embodiment of the invention; and

FIG. 12 shows a high level block diagram of an information processing system useful for implementing one embodiment of the present invention.

## DETAILED DESCRIPTION

Embodiments of the invention relate to a scalable neural hardware for the noisy-OR model of Bayesian networks. One embodiment provides a neural core circuit comprising a pseudo-random number generator for generating random numbers. The neural core circuit further comprises a plurality of incoming electronic axons, a plurality of neural modules, and a plurality of electronic synapses interconnecting the axons to the neural modules. Each synapse interconnects an axon with a neural module. Each neural module receives incoming spikes from interconnected axons. Each neural module represents a noisy-OR gate. Each neural module spikes probabilistically based on at least one random number generated by the pseudo-random number generator.

In one embodiment, each neural module integrates incoming spikes received from interconnected axons, and maintains at least one configurable probability value. Each probability value maintained in each neural module represents a probability that said neural module integrates an incoming spike. Each neural module computes a spiking probability, wherein the computed spiking probability represents a probability that said neural module generates an outgoing spike. For each neural module, the computed spiking probability is based on

the number of integrated spikes and a probability value maintained in said neural module. Each neural module retrieves a random number from the pseudo-random number generator, and generates an outgoing spike only if the retrieved random number is less than the computed spiking probability.

In another embodiment, each neural module maintains at least one configurable probability value, wherein each probability value maintained in said neural module represents a probability that said neural module integrates an incoming spike. Each neural module receives incoming spikes from interconnected axons. For each incoming spike received, each neural module retrieves a random number from the pseudo-random number generator, and integrates said incoming spike only if the retrieved random number is less than a probability value maintained in said neural module. Each neural module generates an outgoing spike only if the number of integrated spikes exceeds a configurable spiking threshold of said neural module.

In one embodiment, the neural core circuit is organized into a scalable noisy-OR neural network including multiple layers of neural core circuits, wherein outgoing spikes from neural modules of a layer are routed to incoming axons of a subsequent layer.

Another embodiment provides a method for generating spikes in a neural network. The method comprises receiving one or more incoming spikes from one or more incoming axons in a neural network, and, in response to the incoming spikes, probabilistically generating an outgoing spike based on or more random numbers using a noisy-OR gate model.

In one embodiment, the method further comprises integrating incoming spikes received from interconnected axons, maintaining at least one configurable probability value, and computing a spiking probability. Each probability value maintained represents a probability of integrating an incoming spike. The computed spiking probability represents a probability of generating an outgoing spike. The computed spiking probability is based on the number of integrated spikes and a probability value maintained. The method further comprises retrieving a random number; and generating an outgoing spike only if the retrieved random number is less than the computed spiking probability.

In one embodiment, the method further comprises maintaining at least one configurable probability value, and receiving incoming spikes from interconnected axons. Each probability value maintained represents a probability of integrating an incoming spike. The method further comprises, for each incoming spike received, retrieving a random number, and integrating the incoming spike only if the retrieved random number is less than a probability value maintained. The method further comprises, for each neural module, generating an outgoing spike only if the number of integrated spikes exceeds a configurable spiking threshold.

In one embodiment, the neural network is a multi-layered scalable noisy-OR neural network, wherein each outgoing spike generated in a layer is routed to incoming axons of a subsequent layer.

Another embodiment provides a non-transitory computer-useable storage medium for a neural core circuit comprising multiple incoming electronic axons and multiple neural modules, the computer-useable storage medium having a computer-readable program. The program upon being processed on a computer causes the computer to implement interconnecting the axons with the neural modules via a synaptic interconnect network comprising plural electronic synapses, and generating random numbers. Each synapse interconnects an axon with a neural module. Each neural module receives incoming spikes from interconnected axons. Each neural

module spikes probabilistically based on at least one generated random number. Each neural module represents a noisy-OR gate.

In one embodiment, the program upon being processed on a computer causes the computer to further implement, for each neural module, integrating incoming spikes received from interconnected axons, maintaining at least one configurable probability value, computing a spiking probability, retrieving a random number, and generating an outgoing spike only if the retrieved random number is less than the computed spiking probability. Each probability value maintained in the neural module represents a probability that the neural module integrates an incoming spike. The computed spiking probability represents a probability that the neural module generates an outgoing spike. The computed spiking probability is based on the number of integrated spikes and a probability value maintained in the neural module.

In one embodiment, the program upon being processed on a computer causes the computer to further implement, for each neural module, maintaining at least one configurable probability value, receiving incoming spikes from interconnected axons, for each incoming spike received, retrieving a random number and integrating the incoming spike only if the retrieved random number is less than a probability value maintained in the neural module, and generating an outgoing spike only if the number of integrated spikes exceeds a configurable spiking threshold of the neural module. Each probability value maintained in said neural module represents a probability that said neural module integrates an incoming spike.

In one embodiment, the program upon being processed on a computer causes the computer to further implement organizing the neural core circuit into a scalable noisy-OR neural network including multiple layers of neural core circuits. Outgoing spikes from neural modules of a layer are routed to incoming axons of a subsequent layer.

The term electronic neuron as used herein represents an architecture configured to simulate a biological neuron. An electronic neuron creates connections between processing elements that are roughly functionally equivalent to neurons of a biological brain. As such, a neuromorphic and synaptronic system comprising electronic neurons according to embodiments of the invention may include various processing elements (including computer simulations) that are modeled on biological neurons. Although certain illustrative embodiments of the invention are described herein using electronic neurons comprising electronic circuits, the present invention is not limited to electronic circuits. A neuromorphic and synaptronic system according to embodiments of the invention can be implemented as a neuromorphic and synaptronic architecture comprising circuitry, and additionally as a computer simulation. Indeed, embodiments of the invention can take the form of an entirely hardware embodiment, an entirely software embodiment, or an embodiment containing both hardware and software elements.

Embodiments of the invention provide neurons ("neural modules") that model noisy-OR gates. The noisy-OR neural modules may be used to perform Bayesian computations, such as performing statistical interference, recognizing patterns, and classifying inputs.

FIG. 1 shows a neural core circuit 100, in accordance with an embodiment of the invention. The neural core circuit 100 comprises multiple pre-synaptic axons 15 and multiple post-

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synaptic noisy-OR neural modules **11**. Specifically, the neural core circuit **100** comprises  $K$  axons **15** and  $M$  neural modules **11**, such as axons  $A_1, A_2, A_3, \dots$ , and  $A_K$ , and neural modules  $N_1, N_2, N_3, \dots$ , and  $N_M$ , wherein  $K$  and  $M$  are positive integers. Each axon **15** and each neural module **11** has configurable operational parameters. Each neural module **11** is connected to a corresponding dendrite **16**, wherein said neural module **11** receives incoming firing events (e.g., incoming spikes) via its corresponding dendrite **16**. As shown in FIG. 1, the neural modules  $N_1, N_2, N_3, \dots$ , and  $N_M$  have corresponding dendrites  $D_1, D_2, D_3, \dots$ , and  $D_M$ , respectively.

As described in detail later herein, each neural module **11** includes a neuron computation circuit that represents a noisy-OR gate. A noisy-OR gate is a canonical interaction model used to describe the interaction between multiple  $n$  causes  $X_1, X_2, \dots, X_n$  and their common effect  $Y$ . Each cause  $X_i$  is assumed to be sufficient to cause  $Y$  independent of the presence of other causes. In one embodiment, each neural module **11** shares the same neuron computation circuit (i.e., multiplexed) with its corresponding dendrite **16**.

The neural core circuit **100** further comprises a synaptic crossbar **12** including multiple synapses **31**, multiple rows/axon paths **26**, and multiple columns/dendrite paths **34**. Each synapse **31** communicates firing events between a pre-synaptic axon **15** and a post-synaptic neural module **11**. Specifically, each synapse **31** is located at cross-point junction between an axon path **26** and a dendrite path **34**, such that a connection between the axon path **26** and the dendrite path **34** is made through said synapse **31**. Each axon **15** is connected to an axon path **26**, such that said axon **15** transmits sends firing events to the connected axon path **26**. A corresponding dendrite **16** of each neural module **11** is connected to a dendrite path **34**, such that said neural module **11** receives firing events from the connected dendrite path **34**.

Further, each axon **15** has a corresponding memory unit **10** maintaining two or more bits of information designating an axon type (e.g., excitatory, inhibitory) of said axon **15**. As shown in FIG. 1, the axons  $A_1, A_2, A_3, \dots$ , and  $A_K$  have corresponding memory units  $G_1, G_2, G_3, \dots$ , and  $G_K$ , respectively. The operational parameters of each neural module **11** includes a strength parameter for each axon type. Each spike integrated by a neural module **11** is weighted based on a strength parameter for the axon type of the axon **15** that said spike is received from.

Each synapse **31** has a synaptic weight. The synaptic weights of the synapses **31** may be represented by a weight matrix  $W$ , wherein an element  $W_{ij}$  represents a synaptic weight of a synapse **31** located at row/axon path  $i$  and column/dendrite path  $j$  of the crossbar **12**. In one embodiment, the synapses **31** are binary memory devices. Each synapse **31** can have a weight “0” indicating that said synapse **31** is non-conducting, or a weight “1” indicating that said synapse **31** is conducting. A learning rule such as spike-timing dependent plasticity (STDP) may be applied to update the synaptic weights of the synapses **31**.

In this specification, an axon vector **30** is used to represent the axon activity of every axon **15** of the neural core circuit **100** in a time step. Specifically, each index of the axon vector **30** represents the axon activity of a corresponding axon **15** of the neural core circuit **100**. Each index with a bit-value of “1” indicates that a corresponding axon **15** has received a firing event in the current time step, wherein the firing event received was generated by a neuron in a previous time step. Each index with a bit-value of “0” indicates that a corresponding axon **15** has not received a firing event in the current time step. For example, as shown in FIG. 1, an axon vector **30** with

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values  $\langle 1, 0, 1, \dots, 0 \rangle$  represents that axons  $A_1$  and  $A_3$  have received a firing event in the current time step.

The neural core circuit **100** further comprises an address-event decoder **40**, an address-event encoder **50**, and a lookup table (LUT) **51**. The address-event decoder **40** is configured to receive address-event packets one at a time. Each address-event packet received includes a firing event generated by a neural module **11** in the same, or a different, neural core circuit **100**. Each address-event packet further includes routing information, such as an address of a target incoming axon **15**. The address-event decoder **40** decodes each address-event packet received and delivers the firing event in said address-event packet to the target incoming axon **15**. Upon receiving a firing event, each axon **15** activates the axon path **26** it is connected to, triggering a read of the axon type of said axon **15** and all synaptic weights on the axon path **26**.

In this specification, an output vector **20** is used to represent the neuron activity of every neural module **11** of the neural core circuit **100** in a time step. Specifically, each index of the output vector **20** represents the neuron activity of a corresponding neural module **11** of the neural core circuit **100**. Each index with a bit-value of “1” indicates a firing event generated by a corresponding neural module **11** in the current time step, wherein the firing event will be routed to a target incoming axon **15** in the same, or a different, neural core circuit **100**. Each index with a bit-value of “0” indicates that a corresponding neural module **11** did not receive sufficient input to generate a firing event. For example, as shown in FIG. 1, the output vector **20** with values  $\langle 1, 1, 0, \dots, 1 \rangle$  indicates that neurons  $N_1, N_2$ , and  $N_M$  are active (i.e., generated a firing event) and neuron  $N_3$  is not active in the current time step. Each neuron  $N_1, N_2$ , and  $N_M$  will send an address-event packet including the generated firing event to a target incoming axon **15** in the same, or a different, neural core circuit **100**.

The address-event encoder **50** is configured to receive firing events generated by the neural modules **11**. The LUT **51** is an address routing table configured to determine target incoming axons **15** for firing events generated by the neural modules **11** of the neural core circuit **100**. A target incoming axon **15** may be an incoming axon **15** in the same neural core circuit **100** or a different neural core circuit **100**. The LUT **51** maintains information such as target distance, direction, addresses, and delivery times. The information maintained in the LUT **51** is used to build an address-event packet for each firing event received.

The neural core circuit **100** further comprises a pseudo-random number generator (PRNG) **13**. The multibit output of the PRNG **13** is thresholded to generate random numbers that are either 0 or 1 or can be compared to other values to generate binary spike outputs. Each neural module **11** is connected to the PRNG **13**. As described in detail later herein, in each time step, each neural module **11** draws a random number from the PRNG **13** to implement the probabilistic spiking of said neural module **11**. In another embodiment, each neural module **11** includes its own PRNG **13**.

As shown in FIG. 1, the neural core circuit **100** further comprises a control module (“controller”) **46** that is connected to a clock **49**. The clock **49** produces clock signals used by the controller **46** to generate time-steps. The controller **46** divides each time-step into operational phases in the neural core circuit **100** for neuron updates, etc.

As described in detail later herein, each neural module **11** includes a neuron computation circuit that represents a noisy-OR gate.

FIG. 2 shows a noisy-OR system **150** including only one noisy-OR neural module **11**, in accordance with an embodiment of the invention. The system **150** comprises a set of  $N$



axons **15**, such as axons  $X_1, X_2, \dots, X_{N-1}$ , and  $X_N$ . The system **150** further comprises a neural module **11** labeled as neural module Y in FIG. 2. Multiple weighted synaptic connections **31** interconnect the axons **15** to the neural module Y, wherein each synaptic connection **31** interconnects an axon **15** to the neural module Y. Each synaptic connection **31** has a synaptic weight.

The neural module Y is configured to function as an N-input noisy-OR gate, wherein the set of N axons **15** represent a set of N inputs. As shown in FIG. 2, each synaptic connection **31** interconnecting an axon  $X_i$  to the neural module Y has a corresponding probability value  $r_i$ . For example, a synaptic connection **31** interconnecting the axon  $X_1$  to the neural module Y has a corresponding probability value  $r_1$ , and a synaptic connection **31** interconnecting the axon  $X_N$  to the neural module Y has a corresponding probability value  $r_N$ . For each firing event received from an axon  $X_i$  via a synaptic connection **31**, the neural module Y integrates said firing event with probability  $r_i$ .

FIG. 3 shows a noisy-OR system **200** including multiple noisy-OR neural modules **11**, in accordance with an embodiment of the invention. The system **200** comprises a set of N axons **15**, such as axons  $X_1, X_2, \dots, X_{N-1}$ , and  $X_N$ . The system **200** further comprises a set of M neural modules **11**, such as neurons  $Y_1, Y_2, \dots, Y_{M-1}$ , and  $Y_M$ . Multiple weighted synaptic connections **31** interconnect the axons **15** to the neural modules **11**, wherein each synaptic connection **31** interconnects an axon **15** to a neural module **11**. Each synaptic connection **31** has a synaptic weight. The synaptic weights of the synaptic connections **31** may be represented by an  $N \times M$  matrix W, wherein each synaptic connection **31** interconnecting an axon  $X_i$  to a neural module  $Y_j$  has a corresponding synaptic weight  $W_{ij}$ .

Each neural module **11** is configured to function as an N-input noisy-OR gate, wherein the set of N axons **15** represent a set of N inputs. Each synaptic connection **31** has a corresponding probability. The probabilities of the synaptic connections **31** may be represented by an  $N \times M$  matrix r, wherein each synaptic connection **31** interconnecting an axon  $X_i$  to a neural module  $Y_j$  has a corresponding probability value  $r_{ij}$ . For each firing event received from an axon  $X_i$  via a synaptic connection **31**, each neural module  $Y_j$  integrates said firing event with probability  $r_{ij}$ .

As stated above, embodiments of the present invention provide neural modules that model noisy-OR gates. In one embodiment, the present invention provides a neural module comprising a neuron computation circuit configured for computing an exponential function. In another embodiment, the present invention provides a neural module comprising a neuron computation circuit that includes a dendrite gate.

FIG. 4 is a block diagram showing a neuron computation circuit **400** for a neural module **11**, wherein the neuron computation circuit **400** is configured to compute an exponential function, in accordance with an embodiment of the invention. In one embodiment, each neural module **11** comprises the

neuron computation circuit **400**. The circuit **400** comprises an integrator unit ("integrator") **2**, a memory unit **4**, an exponential function unit **5**, and a spike check unit **6**.

In each time step, the integrator **2** of each neural module **11** is configured to receive synaptic inputs (i.e., incoming spikes or incoming firing events) from axons **15** connected to the neural module **11** via synapses **31**. In one embodiment, the synaptic inputs received are binary signals comprising of spikes and non-spikes. A spike is represented by 1, and a non-spike is represented by 0.

The memory unit **4** of each neural module **11** maintains different programmable probability values r. In one embodiment, the memory unit **4** maintains different programmable probability values r for different axon types (i.e., the probability values r are axon specific). For example, let  $r_i$  denote the probability a neural module **11** integrates a synaptic input received from an axon **15** with axon type i. If an axon type is denoted as 0, 1, 2, or 3 to differentiate connections with different efficacies, each neural module **11** may maintain different programmable probability values  $r_0, r_1, r_2$ , and  $r_3$  for the different axon types 0, 1, 2, and 3, respectively. In another embodiment, the memory unit **4** maintains different programmable probability values r for different synaptic connections **31** (i.e., the probability values r are synapse specific) or different dendrites **16** (i.e., the probability values r are dendrite specific).

The integrator **2** is further configured to integrate each synaptic input received. Specifically, for each input received via a synapse **31**, the integrator **2** integrates said input only if said input is a spike and the synapse **31** is a conducting synapse. Let n denote the number of inputs integrated by the integrator **2** in a time step.

In this specification, the probability that a neural module **11** spikes ("spiking probability") is denoted as  $P_{\text{spk}}$ . In each time step, the exponential function unit **5** is configured to compute  $P_{\text{spk}}$  only if n is greater than 0. The exponential function unit **5** may compute  $P_{\text{spk}}$  using the following example formula:

$$P_{\text{spk}} = 1 - e^{-(n^{\alpha} r)}$$

In each time step, the spike check unit **6** is configured to draw/retrieve a random number S from the PRNG **13**. The spike check unit **6** determines if the random number S drawn is less than  $P_{\text{spk}}$ . The neural module **11** generates and sends out an outgoing spike only if the random number S drawn is less than  $P_{\text{spk}}$ . n is reset to zero after the neural module **11** spikes.

In another embodiment, the circuit **400** may further comprise a leak unit configured to apply a probabilistic positive leak rate to n so that the neural module **11** spikes with some probability even if all inputs received are 0 values.

Table 1 below provides example pseudo code demonstrating a sequence of operations for implementing probabilistic spiking in a  $j^{\text{th}}$  neural module **11** in conjunction with the neuron computation circuit **400** in FIG. 4.

TABLE 1

---

```

For i=1:N,    //N is the number of axons in the neural core circuit
  If  $A_i=1$  &  $W_{ij}=1$ ,    //the  $i^{\text{th}}$  axon fired & the synapse connecting the  $i^{\text{th}}$  axon and the  $j^{\text{th}}$ 
                        //neural module is a conducting synapse
    n=n+1;    //Increment the number of integrated inputs, n
  Endif;
Endfor;
If n=0,
  No spike;
Else //n>0
  Compute  $P_{\text{spk}} = 1 - e^{-(n^{\alpha} r)}$ ;    //P_spk is the probability that the  $j^{\text{th}}$  neural module spikes

```

---

TABLE 1-continued

---

```

Draw a uniform random number S;    //S is between 0 and 1
If S<P_spk,                        //Compare S to P_spk
    Send out a spike;              //The jth neural module generates a spike only if S<P_spk
Endif;
Endif;

```

---

FIG. 5 is a block diagram showing a neuron computation circuit 450 for a noisy-OR neural module 11, wherein the neuron computation circuit 450 includes a dendrite gate unit 14, in accordance with an embodiment of the invention. In another embodiment, each neural module 11 comprises the neuron computation circuit 450. The circuit 450 comprises the dendrite gate unit (“dendrite gate”) 14, a memory unit 4, an integrator unit (“integrator”) 2, and a threshold check unit 9.

In each time step, the dendrite gate 14 of a neural module 11 is configured to receive synaptic inputs (i.e., incoming spikes or incoming firing events) from axons 15 connected to the neural module 11 via synapses 31. In one embodiment, the synaptic inputs received are binary signals comprising of spikes and non-spikes. A spike is represented by 1, and a non-spike is represented by 0.

The memory unit 4 of each neural module 11 maintains different programmable probability values  $r$ . In one embodiment, the memory unit 4 maintains different programmable probability values  $r$  for each different axon type (i.e., the probability values  $r$  are axon specific). In another embodiment, the memory unit 4 maintains different programmable

the dendrite gate 14 transmits a 1-bit value to the integrator 2 if the random number  $S$  drawn is less than the probability value  $r$ . The dendrite gate 14 transmits a 0-bit value to the integrator 2 if the random number  $S$  drawn reaches or exceeds the probability value  $r$ . As such, the integrator 2 integrates a spike only if a random number  $S$  drawn for the spike is less than the probability value  $r$ . Let  $n$  denote the number of inputs integrated by the integrator 2 in a time step.

Each neural module 11 has a programmable spiking threshold  $V_{th}$ , wherein  $V_{th}$  is a positive integer. In one embodiment, the spiking threshold  $V_{th}$  of each neural module 11 is set to 1, such that said neural module 11 generates and sends out an outgoing spike if the integrator 2 integrates at least one input. Specifically, the threshold check unit 9 of each neural module 11 is configured to determine if  $n$  is greater than zero. If  $n$  is zero, the threshold check unit 9 will not generate a spike. If  $n$  is greater than zero, the threshold check unit 9 generates and sends out an outgoing spike.

Table 2 below provides example pseudo code demonstrating a sequence of operations for implementing probabilistic spiking in a  $j^{th}$  neural module 11 in conjunction with the neuron computation circuit 450 in FIG. 5, wherein the spiking threshold  $V_{th}$  of the  $j^{th}$  neural module 11 is set to 1.

TABLE 2

---

```

For i=1:N,    //N is the number of axons in the neural core circuit
    If Ai=1 & Wij=1,    //the ith axon fired & the synapse connecting the ith axon and the jth
                        //neural module is a conducting synapse
        Draw a uniform random number S;    //S is between 0 and 1
        If S<r,    //Compare S to the probability r maintained in the jth neural module
            n=n+1;    //Increment the number of integrated inputs, n
        Endif;
    Endif;
Endfor;
If n<=0,
    No spike;
Else
    Send out a spike;    //The jth neural module generates a spike only if n>0
Endif;

```

---

probability values  $r$  for different synaptic connections 31 (i.e., the probability values  $r$  are synapse specific). In another embodiment, the memory unit 4 maintains different programmable probability values  $r$  for different dendrites 16 (i.e., the probability values  $r$  are dendrite specific). Each probability value  $r$  maintained denotes the probability that the neural module 11 integrates a synaptic input received.

The dendrite gate 14 includes a comparator component (“comparator”) 14B. For each spike received via a conducting synapse 31, the dendrite gate 14 is further configured to draw a random number  $S$  from the PRNG 13, use the comparator 14B to determine whether the random number  $S$  drawn is less than a probability value  $r$  maintained in the memory unit 14, and transmit a binary signal to the integrator 2. Specifically,

In another embodiment, the spiking threshold  $V_{th}$  of each neural module 11 is greater than 1. As such, the threshold check unit 9 of each neural module 11 is configured to determine whether the number of integrated inputs  $n$  exceeds the spiking threshold  $V_{th}$  of said neural module 11. Specifically, if  $n$  is less than or equal to  $V_{th}$ , the threshold check unit 9 will not generate a spike. If  $n$  exceeds  $V_{th}$ , the threshold check unit 9 generates and sends out an outgoing spike.

Table 3 below provides example pseudo code demonstrating a sequence of operations for implementing probabilistic spiking in a  $j$  neural module 11 in conjunction with the neuron computation circuit 450 in FIG. 5, wherein the spiking threshold  $V_{th}$  of the  $j^{th}$  neural module 11 is greater than 1.

TABLE 3

---

```

For i=1:N,      //N is the number of axons in the neural core circuit
  If Ai==1 & Wij==1,    //the ith axon fired & the synapse connecting the ith axon and the jth
                        //neural module is a conducting synapse
    Draw a uniform random number S;      //S is between 0 and 1
    If S<r,    //Compare S to the probability r maintained in the jth neural module
        n=n+1;    //Increment the number of integrated inputs, n
    Endif;
  Endif;
Endfor;
If n<=Vth, //Compare n to the spiking threshold Vth of the jth neural module
  No spike;
Else
  Send out a spike;    //The jth neural module generates a spike only if n>Vth
Endif;

```

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In another embodiment, the circuit **450** may further comprise a leak unit configured to apply a probabilistic positive leak rate to  $n$  so that the neural module **11** spikes with some probability even if all inputs received are 0.

FIG. 6 shows a scalable noisy-OR neural network **500**, in accordance with an embodiment of the invention. The number of inputs received by a neural module **11** is limited by the size of the crossbar **12** of the neural core circuit **100** containing the neural module **11**. For example, for a neural core circuit **100** having only 256 incoming axons, the number of inputs that a neural module **11** of the neural core circuit **100** can receive is limited to 256.

Each neural module **11** in the network **500** receives more inputs than a neural core circuit **100** containing said neural module **11** is capable of receiving. To overcome the size limitations of each individual neural core circuit **100**, the network **500** may be implemented by organizing multiple neural core circuits **100** into multiple layers **501** of neural core circuits **100**. Each layer **501** comprises at least one neural core circuit **100**. Each neural module **11** of a first layer **501** (e.g., First Layer) is configured to model a noisy-OR gate, wherein the output (e.g., spike) generated is based on more than 256 inputs. Each neural module **11** of a second layer **501** (e.g., Second Layer) or an intermediate layer **501** is configured to model a pure OR gate. The second layer **501** and the intermediate layers **501** (i.e., the subsequent layers **501** after the first layer **501**) are configured to integrate all input received. Output from one layer **501** are routed to a subsequent layer **501** using address-event packets.

FIG. 7 shows an example graph **600** plotting the spiking probabilities of different example noisy-OR neural modules **11**, wherein each neural module **11** has a different probability value  $r$ , in accordance with an embodiment of the invention. Each curve **601**, **602**, **603**, and **604** represents a neural module **11** having a spiking threshold  $V_{th}$  set to 1. As such, each neural module **11** represented in the graph **600** generates an outgoing spike if the number of integrated inputs  $n$  in said neural module **11** is greater than zero. Increasing the probability value  $r$  of a neural module **11** sharpens the slope of the curve representing the neural module **11**.

FIG. 8 shows an example graph **650** plotting the spiking probabilities of different example noisy-OR neural modules **11**, wherein each neural module **11** has a different spiking threshold  $V_{th}$ , in accordance with an embodiment of the invention. Each curve **651**, **652**, **653**, and **654** represents a neural module **11** having a spiking threshold  $V_{th}$  that is greater than 1. As such, each curve **651**, **652**, **653**, and **654** represents a noisy sigmoid. Sigmoids are utilized in multiple types of neural and Bayesian network applications, such as Restricted Boltzmann machines.

Increasing the spiking threshold  $V_{th}$  of a neural module **11** shifts the sigmoid representing the neural module **11** further to the right. For example, the curve **651** represents a neural module **11** having a spiking threshold  $V_{th}$  that is greater than a different spiking threshold  $V_{th}$  maintained in a neural module **11** that is represented by the curve **654**.

FIG. 9 shows an example graph **700** plotting the spiking probabilities of different example noisy-OR neural modules **11**, wherein each neural module **11** has a different spiking threshold  $V_{th}$  and maintains a different probability value  $r$ , in accordance with an embodiment of the invention. Each curve **701**, **702**, **703**, and **704** represents a noisy sigmoid. Increasing the probability value  $r$  of a neural module **11** sharpens the slope of the sigmoid representing the neural module **11**. The curves intersect at reference point **705**. The reference point **705** also indicates the half-way point of each curve. As such, the half-way point of a curve is preserved as the slope of the curve changes.

FIG. 10 is a flowchart of an example process **800** for implementing probabilistic spiking in a neural module, wherein the process **800** includes computing an exponential function, in accordance with an embodiment of the invention. In process block **801**, synaptic inputs (e.g., incoming spikes) are integrated, wherein the number of inputs integrated is denoted as  $n$ . In process block **802**, whether  $n$  is greater than zero is determined. If  $n$  is equal to zero, return to process block **801**. If  $n$  is greater than zero, the probability that the neural module will spike ("P\_spk") is determined, as shown in process block **803**. In one example implementation, P\_spk is determined by computing  $1 - e^{-(n \cdot r)}$ . In process block **804**, a random number  $S$  is drawn (e.g., from a pseudo-random number generator). In process block **805**, whether  $S$  is less than P\_spk is determined. If  $S$  is less than P\_spk, proceed to process block **806** where the neural module generates and sends an outgoing spike. If  $S$  is equal to or greater than P\_spk, return to process block **801**.

FIG. 11 is a flowchart of an example process **900** for implementing probabilistic spiking in a neural module, wherein the process **900** includes determining whether the number of inputs integrated in the neural module is greater than a spiking threshold of the neural module, in accordance with an embodiment of the invention. In process block **901**, whether a synaptic input (e.g., a spike) is received is determined. If an input is received, a random number  $S$  is drawn (e.g., from a pseudo-random number generator) as shown in process block **902**. If no input is received, return to process block **901**. In process block **903**, whether  $S$  is less than a probability value  $r$  maintained in the neural module is determined. If  $S$  is less than  $r$ , the neural module integrates the received input by incrementing  $n$ , wherein  $n$  represents the number of integrated inputs, as shown in process block **904**. If

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S is equal to or greater than r, the neural module ignores the received input and returns to process block 901.

In process block 905, whether n is greater than a spiking threshold  $V_{th}$  of the neural module is determined. If n is greater than  $V_{th}$ , the neural module generates and sends an outgoing spike as shown in process block 906. If n is less than or equal to  $V_{th}$ , return to process block 901.

FIG. 12 is a high level block diagram showing an information processing system 300 useful for implementing one embodiment of the present invention. The computer system includes one or more processors, such as processor 302. The processor 302 is connected to a communication infrastructure 304 (e.g., a communications bus, cross-over bar, or network).

The computer system can include a display interface 306 that forwards graphics, text, and other data from the communication infrastructure 304 (or from a frame buffer not shown) for display on a display unit 308. The computer system also includes a main memory 310, preferably random access memory (RAM), and may also include a secondary memory 312. The secondary memory 312 may include, for example, a hard disk drive 314 and/or a removable storage drive 316, representing, for example, a floppy disk drive, a magnetic tape drive, or an optical disk drive. The removable storage drive 316 reads from and/or writes to a removable storage unit 318 in a manner well known to those having ordinary skill in the art. Removable storage unit 318 represents, for example, a floppy disk, a compact disc, a magnetic tape, or an optical disk, etc. which is read by and written to by removable storage drive 316. As will be appreciated, the removable storage unit 318 includes a computer readable medium having stored therein computer software and/or data.

In alternative embodiments, the secondary memory 312 may include other similar means for allowing computer programs or other instructions to be loaded into the computer system. Such means may include, for example, a removable storage unit 320 and an interface 322. Examples of such means may include a program package and package interface (such as that found in video game devices), a removable memory chip (such as an EPROM, or PROM) and associated socket, and other removable storage units 320 and interfaces 322 which allow software and data to be transferred from the removable storage unit 320 to the computer system.

The computer system may also include a communication interface 324. Communication interface 324 allows software and data to be transferred between the computer system and external devices. Examples of communication interface 324 may include a modem, a network interface (such as an Ethernet card), a communication port, or a PCMCIA slot and card, etc. Software and data transferred via communication interface 324 are in the form of signals which may be, for example, electronic, electromagnetic, optical, or other signals capable of being received by communication interface 324. These signals are provided to communication interface 324 via a communication path (i.e., channel) 326. This communication path 326 carries signals and may be implemented using wire or cable, fiber optics, a phone line, a cellular phone link, an RF link, and/or other communication channels.

In this document, the terms "computer program medium," "computer usable medium," and "computer readable medium" are used to generally refer to media such as main memory 310 and secondary memory 312, removable storage drive 316, and a hard disk installed in hard disk drive 314.

Computer programs (also called computer control logic) are stored in main memory 310 and/or secondary memory 312. Computer programs may also be received via communication interface 324. Such computer programs, when run, enable the computer system to perform the features of the

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present invention as discussed herein. In particular, the computer programs, when run, enable the processor 302 to perform the features of the computer system. Accordingly, such computer programs represent controllers of the computer system.

From the above description, it can be seen that the present invention provides a system, computer program product, and method for implementing the embodiments of the invention. The present invention further provides a non-transitory computer-useable storage medium for neuromorphic event-driven neural computing in a scalable neural network. The non-transitory computer-useable storage medium has a computer-readable program, wherein the program upon being processed on a computer causes the computer to implement the steps of the present invention according to the embodiments described herein. References in the claims to an element in the singular is not intended to mean "one and only" unless explicitly so stated, but rather "one or more." All structural and functional equivalents to the elements of the above-described exemplary embodiment that are currently known or later come to be known to those of ordinary skill in the art are intended to be encompassed by the present claims. No claim element herein is to be construed under the provisions of 35 U.S.C. section 112, sixth paragraph, unless the element is expressly recited using the phrase "means for" or "step for."

The terminology used herein is for the purpose of describing particular embodiments only and is not intended to be limiting of the invention. As used herein, the singular forms "a," "an" and "the" are intended to include the plural forms as well, unless the context clearly indicates otherwise. It will be further understood that the terms "comprises" and/or "comprising," when used in this specification, specify the presence of stated features, integers, steps, operations, elements, and/or components, but do not preclude the presence or addition of one or more other features, integers, steps, operations, elements, components, and/or groups thereof.

The corresponding structures, materials, acts, and equivalents of all means or step plus function elements in the claims below are intended to include any structure, material, or act for performing the function in combination with other claimed elements as specifically claimed. The description of the present invention has been presented for purposes of illustration and description, but is not intended to be exhaustive or limited to the invention in the form disclosed. Many modifications and variations will be apparent to those of ordinary skill in the art without departing from the scope and spirit of the invention. The embodiment was chosen and described in order to best explain the principles of the invention and the practical application, and to enable others of ordinary skill in the art to understand the invention for various embodiments with various modifications as are suited to the particular use contemplated.

What is claimed is:

1. A neural core circuit comprising:

a pseudo-random number generator (PRNG) for generating random numbers;

a plurality of incoming electronic axons;

a plurality of neural modules, wherein each neural module maintains one or more configurable probability values, wherein each neural module integrates incoming spikes probabilistically based in part on at least one random number generated by the PRNG and at least one of said probability values, wherein each neural module generates outgoing spikes probabilistically based in part on a comparison between at least one random number gen-

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erated by the PRNG and at least one of said probability values, and wherein each neural module represents a noisy-OR gate; and  
 a plurality of electronic synapses interconnecting the axons with the neural modules, wherein each synapse interconnects an axon with a neural module, and wherein each neural module receives incoming spikes from interconnected axons.

2. The neural core circuit of claim 1, wherein:  
 each neural module integrates incoming spikes received from interconnected axons;  
 each probability value maintained in each neural module represents a probability that said neural module integrates an incoming spike; and  
 each neural module computes a spiking probability of said neural module, wherein the computed spiking probability represents a probability that said neural module generates an outgoing spike.

3. The neural core circuit of claim 2, wherein:  
 for each neural module:  
 the computed spiking probability is based on the number of integrated spikes and at least one of said probability values maintained in said neural module.

4. The neural core circuit of claim 3, wherein each neural module:  
 retrieves a random number from the (PRNG); and  
 generates an outgoing spike only if the retrieved random number is less than the computed spiking probability.

5. The neural core circuit of claim 1, wherein:  
 each probability value maintained in said neural module represents a probability that said neural module integrates an incoming spike; and  
 each neural module receives incoming spikes from interconnected axons.

6. The neural core circuit of claim 5, wherein:  
 for each incoming spike received, each neural module:  
 retrieves a random number from the (PRNG); and  
 integrates said incoming spike only if the retrieved random number is less than a probability value maintained in said neural module.

7. The neural core circuit of claim 6, wherein each neural module:  
 generates an outgoing spike only if the number of integrated spikes exceeds a configurable spiking threshold of said neural module.

8. The neural core circuit of claim 1, wherein:  
 the neural core circuit is organized into a scalable noisy-OR neural network including multiple layers of neural core circuits, wherein outgoing spikes from neural modules of a layer are routed to incoming axons of a subsequent layer.

9. A method of generating spikes in a neural network, comprising:  
 receiving one or more incoming spikes from one or more incoming axons in a neural network;  
 maintaining one or more configurable probability values; and  
 in response to said one or more incoming spikes, for each neuron module:  
 probabilistically integrating the incoming spikes based in part on a comparison between at least one random number generated by a pseudo-random number generator (PRNG) and at least one of said probability values using a noisy-OR gate model; and

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probabilistically generating an outgoing spike based in part on at least one random number generated by the PRNG and at least one of said probability values using the noisy-OR gate model.

10. The method of claim 9, further comprising:  
 integrating incoming spikes received from interconnected axons; and  
 computing a spiking probability, wherein the computed spiking probability represents a probability of generating an outgoing spike;  
 wherein each probability value maintained represents a probability of integrating an incoming spike.

11. The method of claim 10, wherein:  
 the computed spiking probability is based on the number of integrated spikes and at least one of said probability values.

12. The method of claim 11, further comprising:  
 retrieving a random number from the PRNG; and  
 generating an outgoing spike only if the retrieved random number is less than the computed spiking probability.

13. The method of claim 9, further comprising:  
 receiving incoming spikes from interconnected axons;  
 wherein each probability value maintained represents a probability of integrating an incoming spike.

14. The method of claim 13, further comprising:  
 for each incoming spike received:  
 retrieving a random number from the PRNG; and  
 integrating said incoming spike only if the retrieved random number is less than a probability value maintained.

15. The method of claim 14, further comprising:  
 for each neural module, generating an outgoing spike only if the number of integrated spikes exceeds a configurable spiking threshold.

16. The method of claim 9, wherein:  
 the neural network is a multi-layered scalable noisy-OR neural network, wherein each outgoing spike generated in a layer is routed to incoming axons of a subsequent layer.

17. A non-transitory computer-useable storage medium for a neural core circuit comprising multiple incoming electronic axons and multiple neural modules, the computer-useable storage medium having a computer-readable program, wherein the program upon being processed on a computer causes the computer to implement:  
 interconnecting the axons with the neural modules via a synaptic interconnect network comprising plural electronic synapses, wherein each synapse interconnects an axon with a neural module, and wherein each neural module receives incoming spikes from interconnected axons; and  
 generating random numbers;  
 wherein each neural module maintains one or more configurable probability values, wherein each neural module integrates incoming spikes probabilistically based in part on a comparison between at least one random number generated by a pseudo-random number generator (PRNG) and at least one of said probability values, wherein each neural module generates outgoing spikes probabilistically based in part on at least one random

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number generated by the PRNG and at least one of said probability values, and wherein each neural module represents a noisy-OR gate.

**18.** The program of claim 17, further causing the computer to implement:

for each neural module:

integrating incoming spikes received from interconnected axons;

computing a spiking probability, wherein the computed spiking probability represents a probability that said neural module generates an outgoing spike;

retrieving a random number; and

generating an outgoing spike only if the retrieved random number is less than the computed spiking probability;

wherein each probability value maintained in said neural module represents a probability that said neural module integrates an incoming spike; and

wherein the computed spiking probability is based on the number of integrated spikes and a probability value maintained in said neural module.

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**19.** The program of claim 17, further causing the computer to implement:

for each neural module:

receiving incoming spikes from interconnected axons;

for each incoming spike received, retrieving a random number and integrating said incoming spike only if the retrieved random number is less than a probability value maintained in said neural module; and

generating an outgoing spike only if the number of integrated spikes exceeds a configurable spiking threshold of said neural module;

wherein each probability value maintained in said neural module represents a probability that said neural module integrates an incoming spike.

**20.** The program of claim 17, further causing the computer to implement:

organizing the neural core circuit into a scalable noisy-OR neural network including multiple layers of neural core circuits, wherein outgoing spikes from neural modules of a layer are routed to incoming axons of a subsequent layer.

\* \* \* \* \*